The following is to be added to Example given in Article 47 (Page 33) of Chapter 4 on "Arithmetic Progression" in Higher Algebra by Hall and Knight (Reprint 1894, Macmillan and Co.)

The example is reproduced below –

Example. How many terms of the series $-9, -6, -3, \ldots$ must be taken that the sum may be 66?

Here $\qquad \frac{n}{2}\{-18+(n-1)\,3\}=66;$

that is, $\qquad n^2-7n-44=0,$

or $\qquad (n-11)\,(n+4)=0;$

$\qquad \therefore \quad n=11 \text{ or } -4.$

If we take 11 terms of the series, we have

$\qquad -9, -6, -3, 0, 3, 6, 9, 12, 15, 18, 21;$

the sum of which is 66.

If we begin at the *last* of these terms and *count backwards* four terms, the sum is also 66; and thus, although the negative solution does not directly answer the question proposed, we are enabled to give it an intelligible meaning, and we see that it answers a question closely connected with that to which the positive solution applies.

Another way of interpreting the result $n = -4$ is as follows. Any arithmetic progression is defined by three things a) its first term b) its common difference and c) the direction in which it propagates. The given series has -9 as its first term and 3 as its common difference and it propagates to the right i.e. succeeding terms are written to the right hand side of the first term. In this way it can also be looked upon as a number line or the X axis of a X – Y coordinate system in which the first term coincides with the origin and subsequent terms are points in the positive direction of X axis / number line. We can shift the origin or the first term of the series to any number of the series we want. The series can also be extended in the negative direction to the left of first term applying the same common difference. Doing this results in the following series –

................,-21, -18,-15,-12,-9,-6,-3,0,3,6,9,12,15,18,21…………..

(The first term of the series is -9 and is the origin of the so implied number line / X axis. The first term can be any term say -12, -18, 6 or 9 and so on.)

Having decided our first term, the result n = -4 means we propagate in the negative direction by 4 terms. The terms in the negative direction are obtained by reversing the sign of the terms of the series given above. So the series in the positive as well as negative direction is as given below –

……………,21, 18,15,12,-9,-6,-3,0,3,6,9,12,15,18,21…………..

(Negative direction of series)   (Positive direction of series)
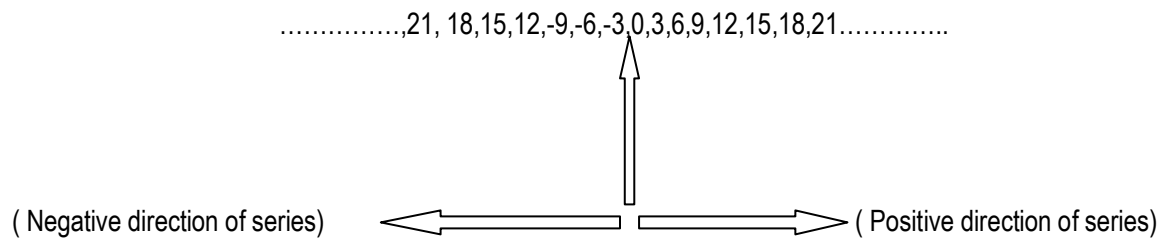
In the series so obtained the sum of 4 terms in the negative direction will add to 66 as it does – 12+15+18+21 = 66. This is how the second result n = - 4 can be interpreted i.e write the series both in positive and negative direction and sum the number of terms in the negative direction. This interpretation of the root n = -4 is more logical than interpretation derived based on other root n = 11.

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May like to consider this as an addendum at the end of the book in next reprint.

Regards

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